Structure and forces in stressed 3D packings

Nicolas Brodu, Joshua Dijksman, Robert Behringer

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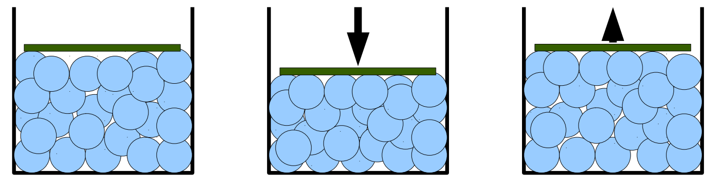
Rationale / Why

Mechanical stresses on granular assemblies are ubiquitous

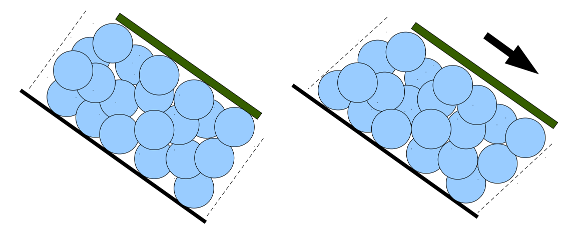
– Repeated compression

(e.g. trucks passing on a road)



– Shearing

(e.g. gravitational pull on a mountain slope)



– Industrial processes, civil engineering, environment...

Understanding their macroscopic response is necessary

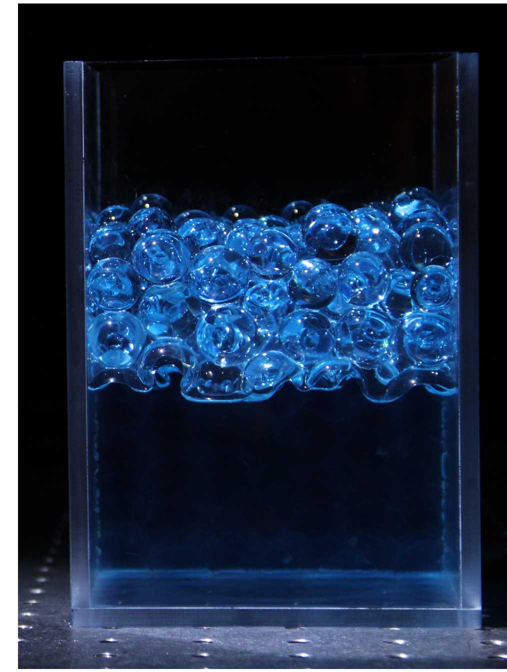
– Requires "seeing" what is the state at the level of grains

Most studies are 2D. Most real cases are 3D.

This work = structure + forces in 3D

Accessing the micro-structure

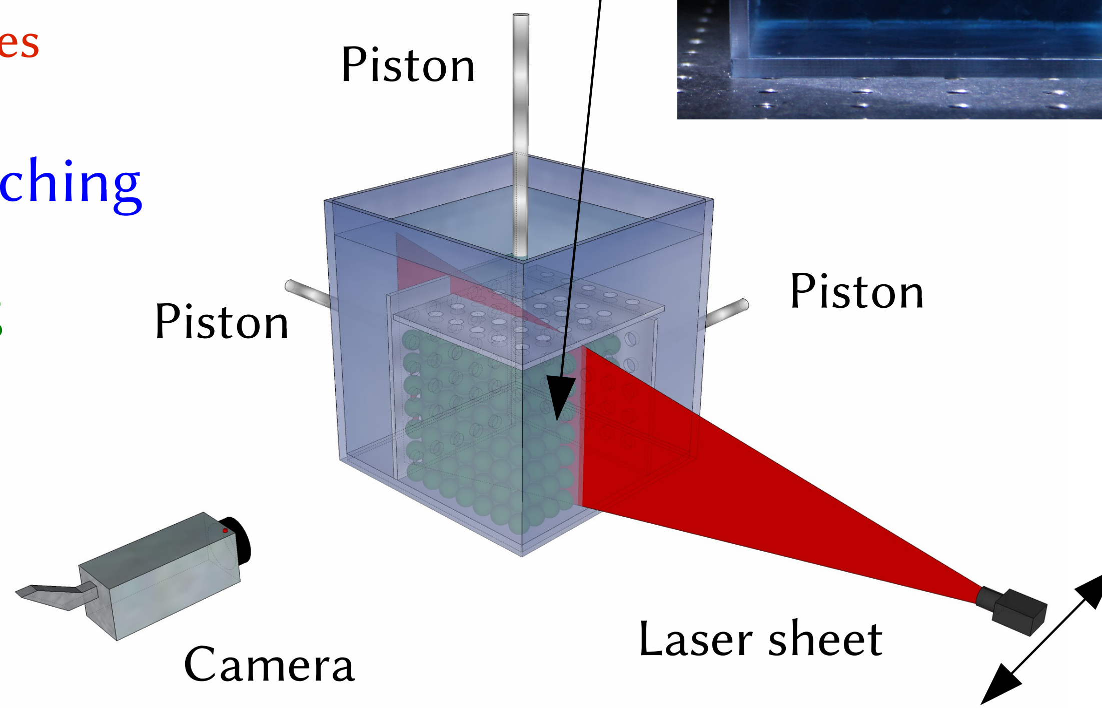
Mukhopadhyay *et al.*
Phys. Rev. E 84, 011302, 2011
Dijksman *et al.*
Rev. Sci. Intstrum. 2012



X-rays / micro-ct
- Fine resolution
- Most materials
- Costly

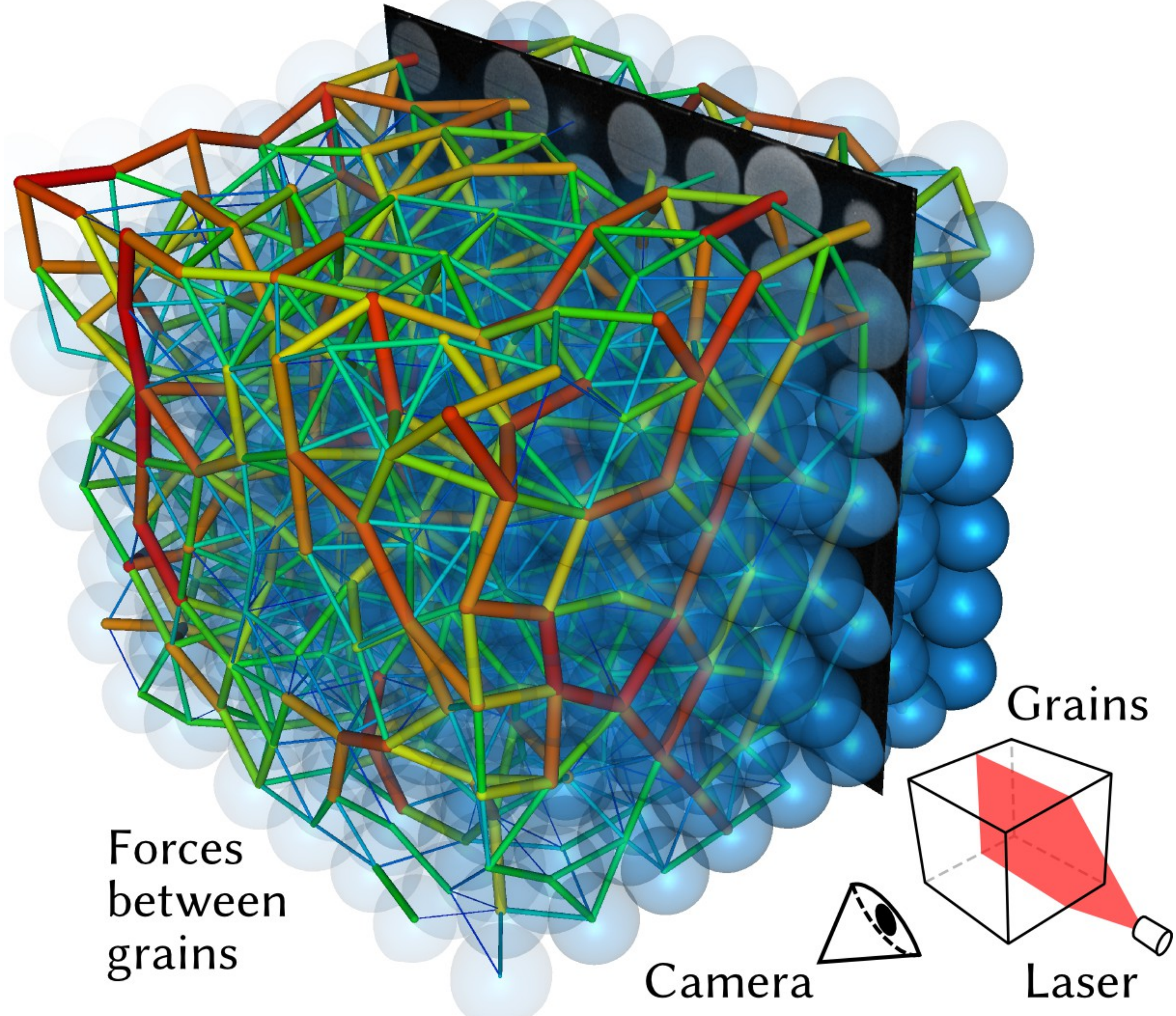
Confocal: emulsions
- Microscopic
- Costly
- Difficult to control applied stresses

This work: refractive index matching
- Macroscopic grains
- Easy to control, tri-axial shearing
- Cheap
- Submersed

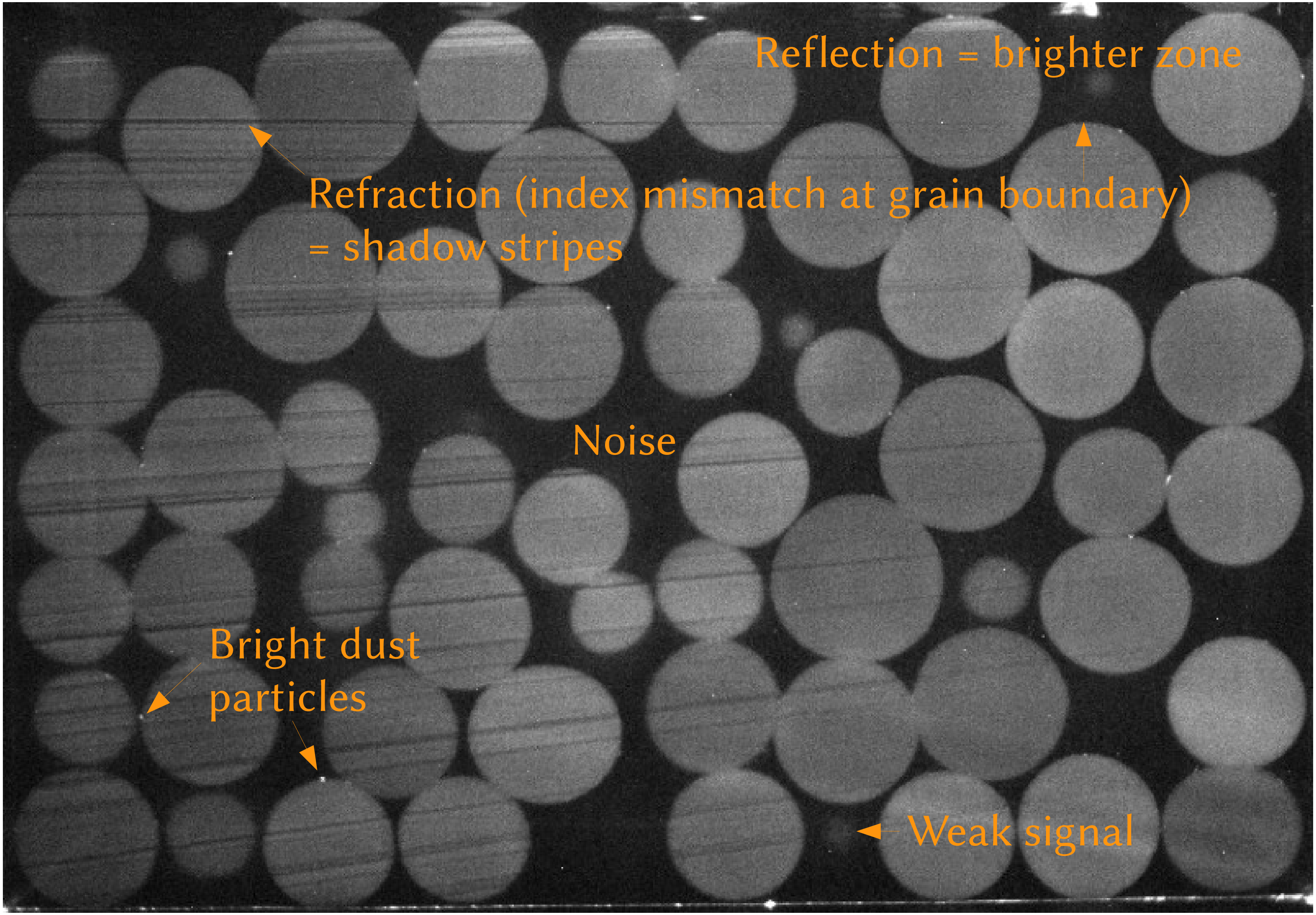


Next slides on:
1. Structure
2. Forces in 3D

What we get

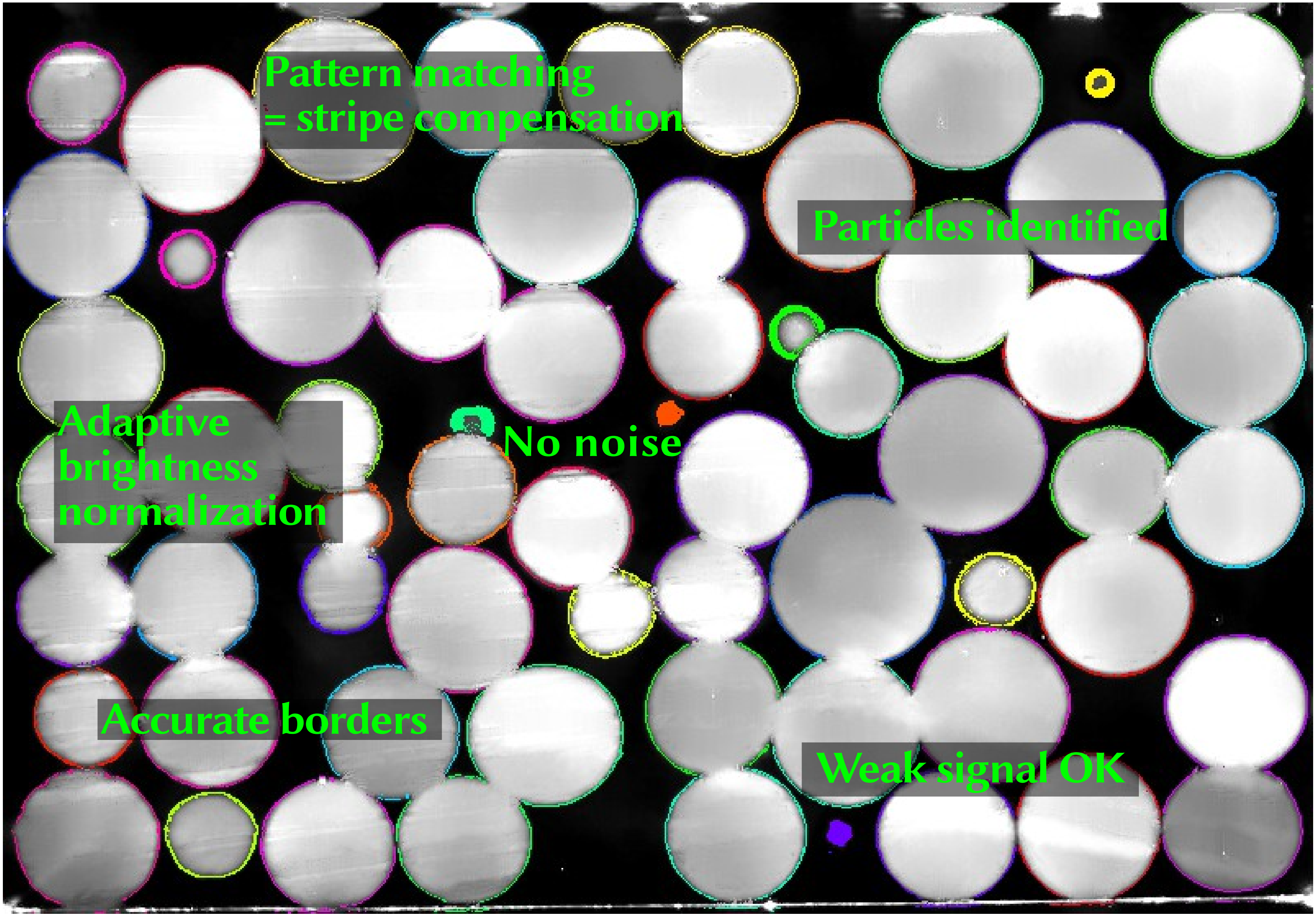


Typical image



Your mission, should you choose to accept it, is to turn this into reliable 3D forces...

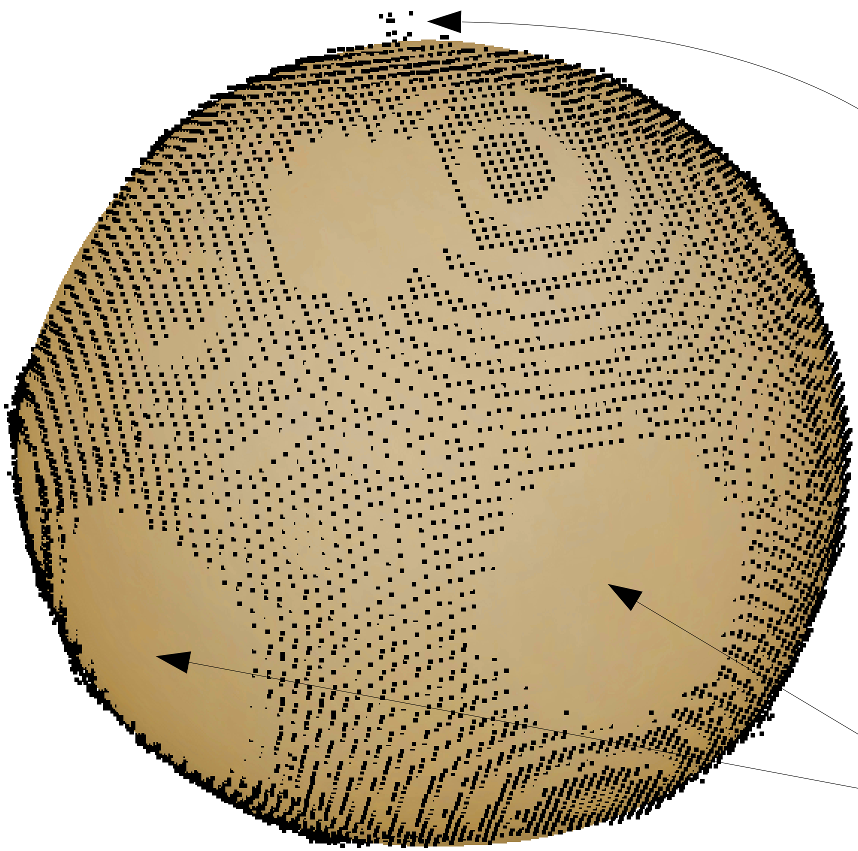
Image processing



Goal here = reliable structure information & reliable shapes ⇒ forces

From 2D images to 3D grains

Step 1: Stack the images into 3D voxel
Step 2: Detect border voxels
Step 3: Fit an analytic surface to these borders
Done here using a spline basis of functions on the unit sphere
Step 4: Use these surfaces to get accurate forces

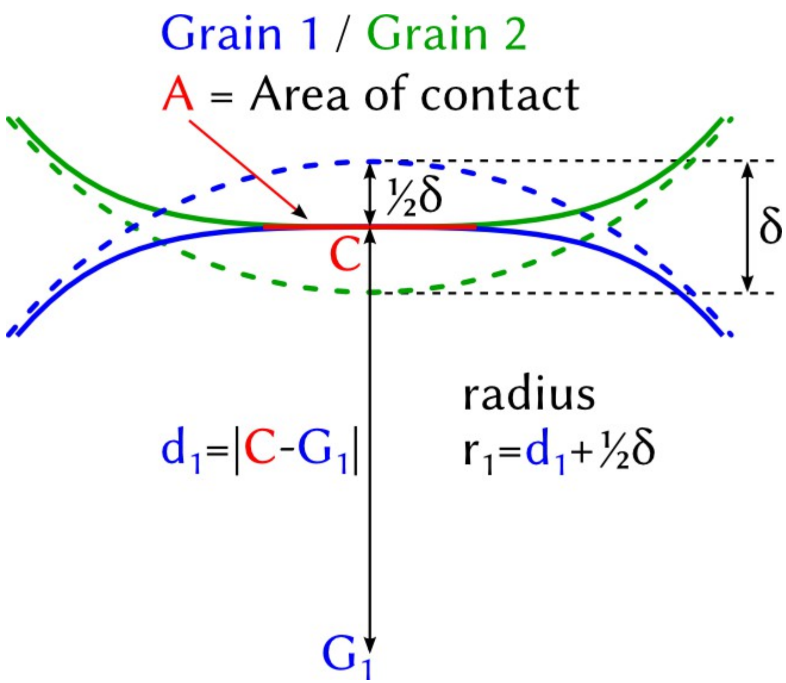


Outliers completely eliminated

Contacts = no border between grains
BUT surface area is well defined

Inferring forces in full 3D

Analytic shape descriptions ⟹ contact properties



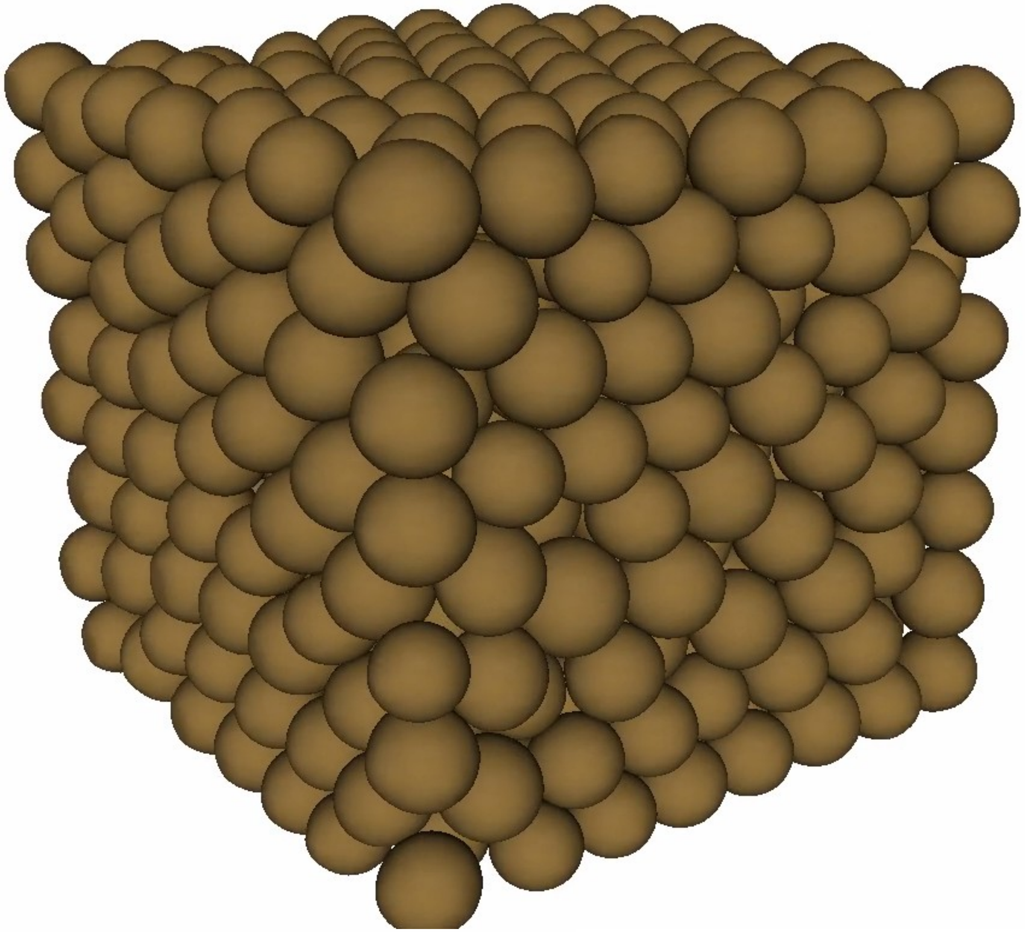
Measured here: A, C, G_1, G_2, d_1, d_2.
Unknown: δ

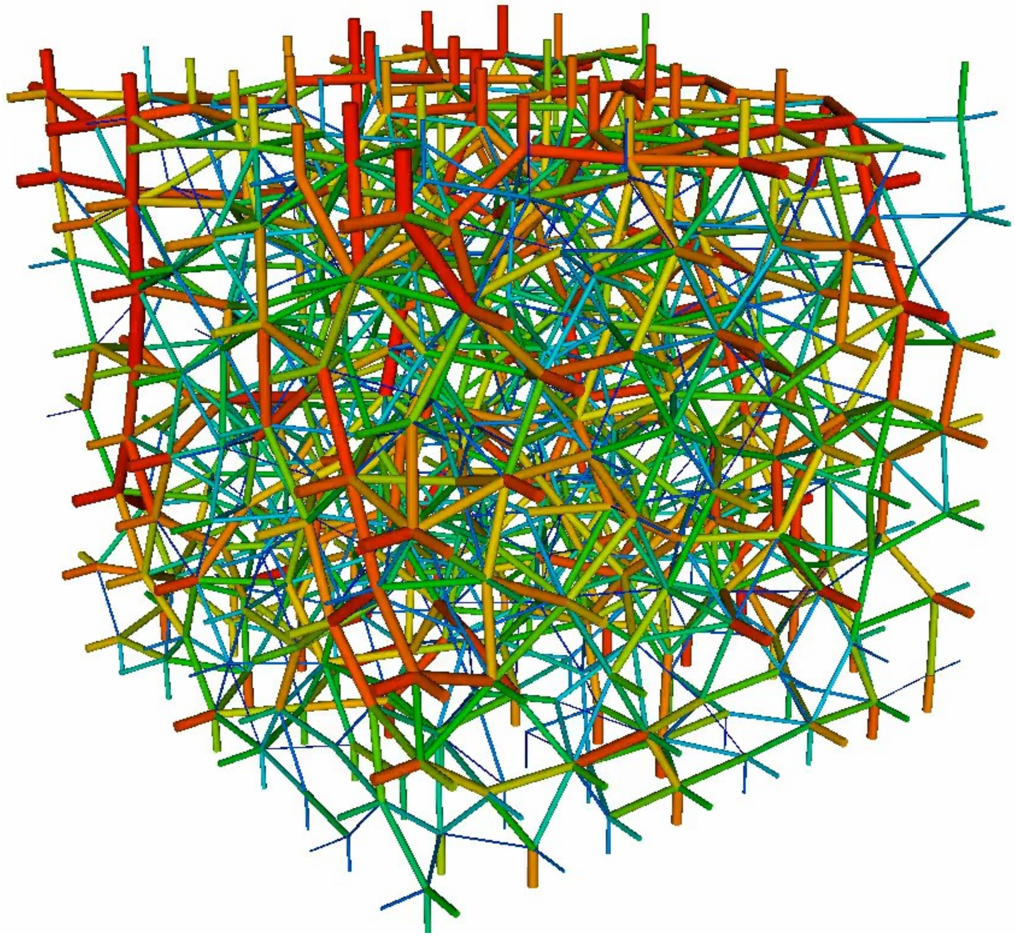
Contact properties ⟹ forces

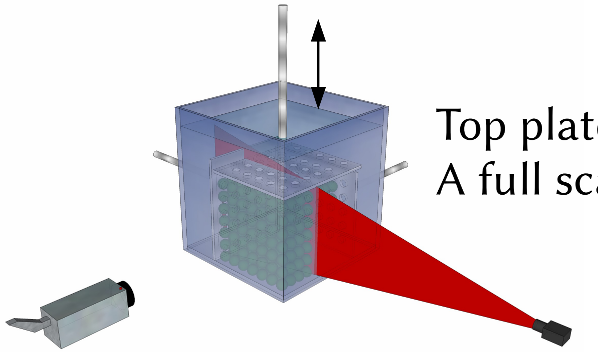
- $1/r = 1/r_1+1/r_2$ radius of curvature at contact
- $F = E\, r^{1/2}\, \delta^{3/2}$ E = effective Young modulus
- $F = E\, \delta\, a$ $a = \sqrt{A/\pi}$ radius of the contact
- Hence $r\,\delta = A/\pi \implies \delta \implies F$ (with given E)

⟹ Vector forces in full 3D, with orientation, position, norm
+ grain centers of mass, stress tensor, etc.

10 UNI-AXIAL COMPRESSION CYCLES



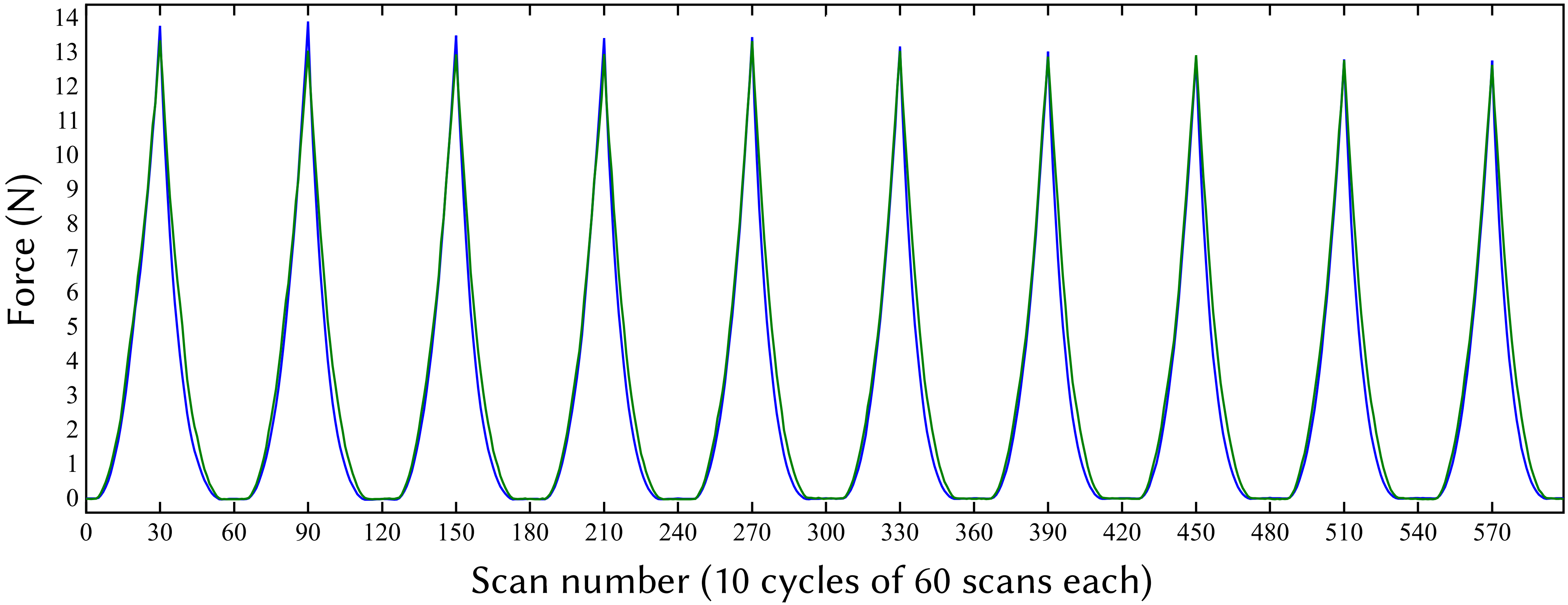




Top plate moves by 1mm increments
A full scan is taken between increments

Forces = struts joining the grain centers
Blue = weakest, Red = strongest

Validation on compression cycles



Blue = force measured on the top plate sensor

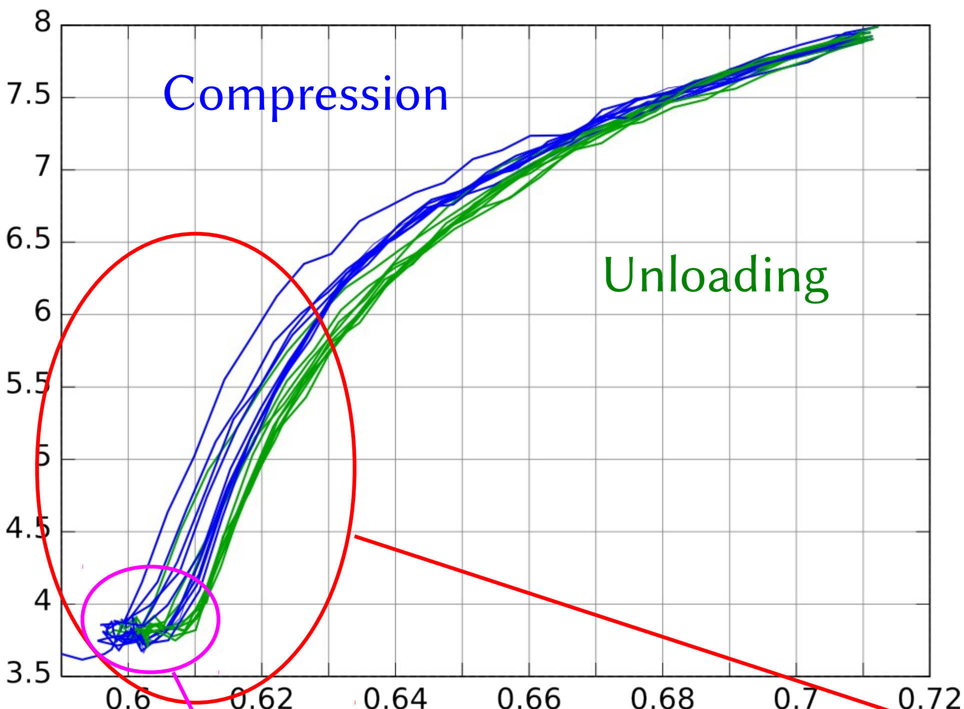
Green = force inferred from the images + measure of E=22.4 kPa

Grain deformation up to ≈13%, scan processed independently: no global fitting

$\approx 980 \cdot 10^3$ contacts over 600 scans. Resolution $\approx 10^{-2}$ N.

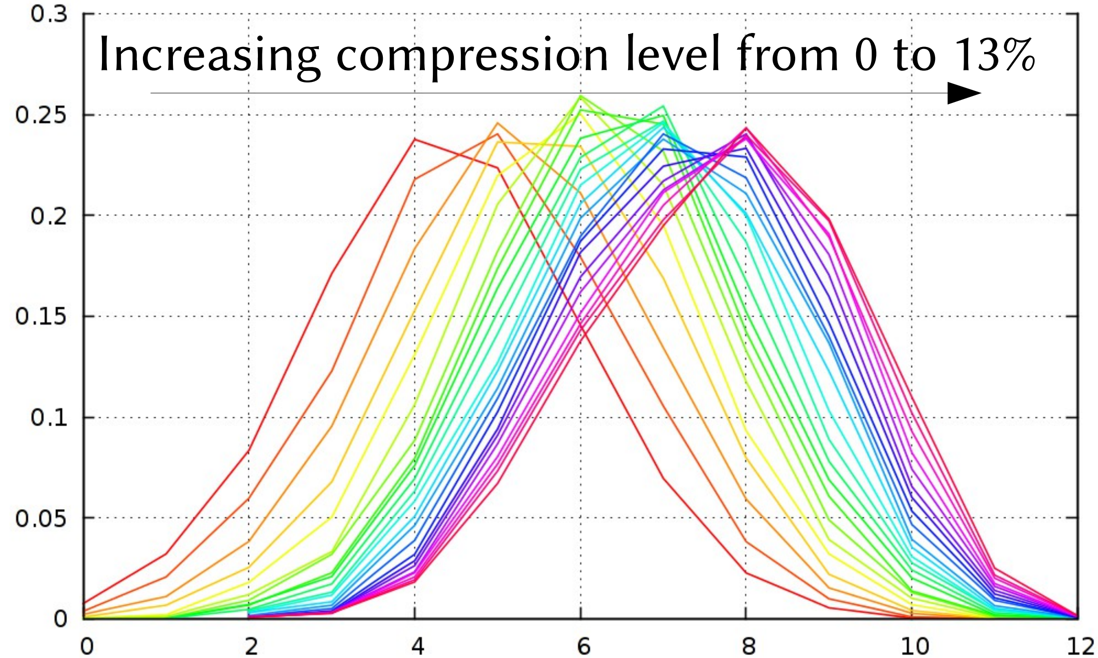
Number of contacts per grain Z

Number of
contacts
per grain (Z)



Volume fraction φ

p(Z)

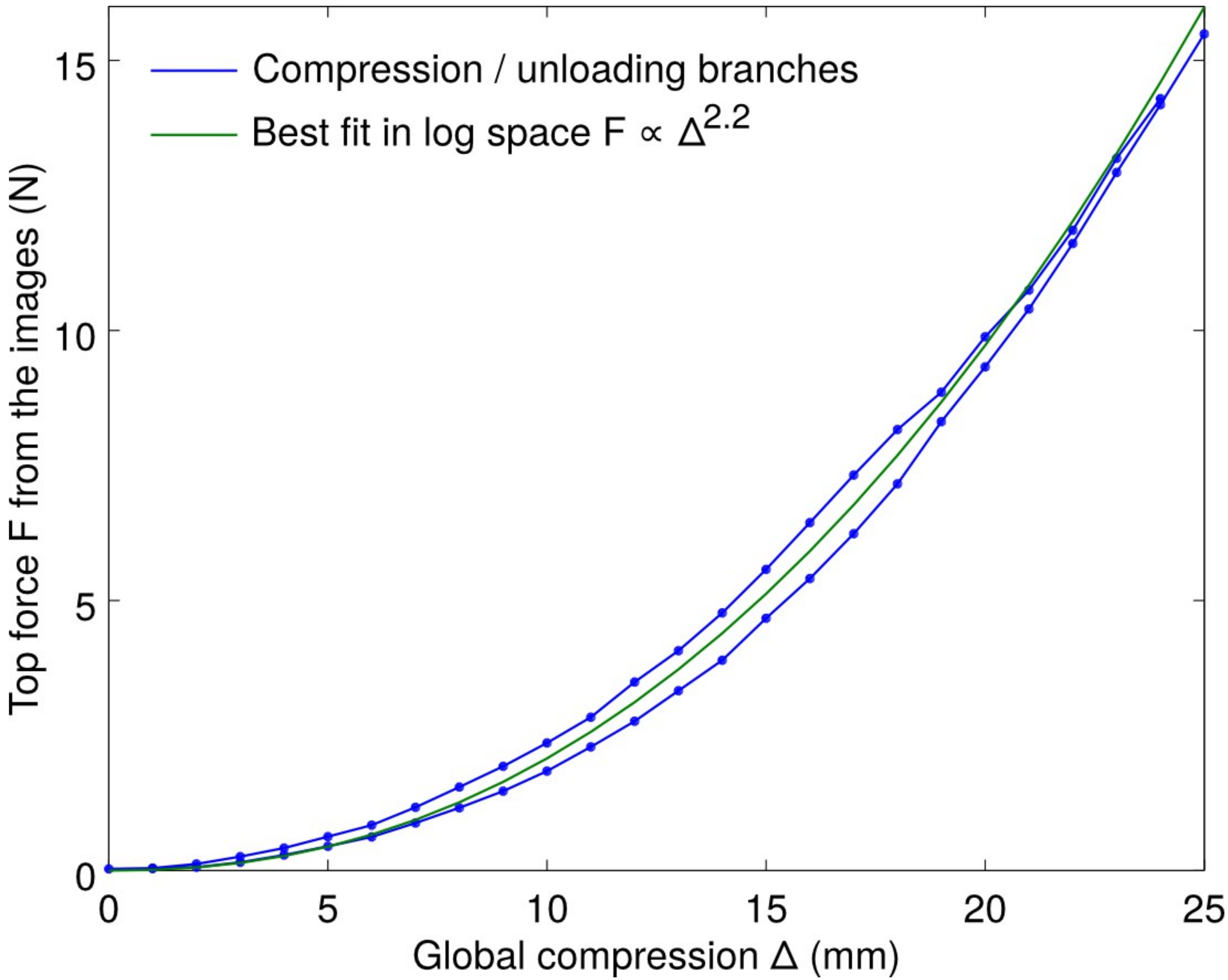


Number of contacts per grain (Z)

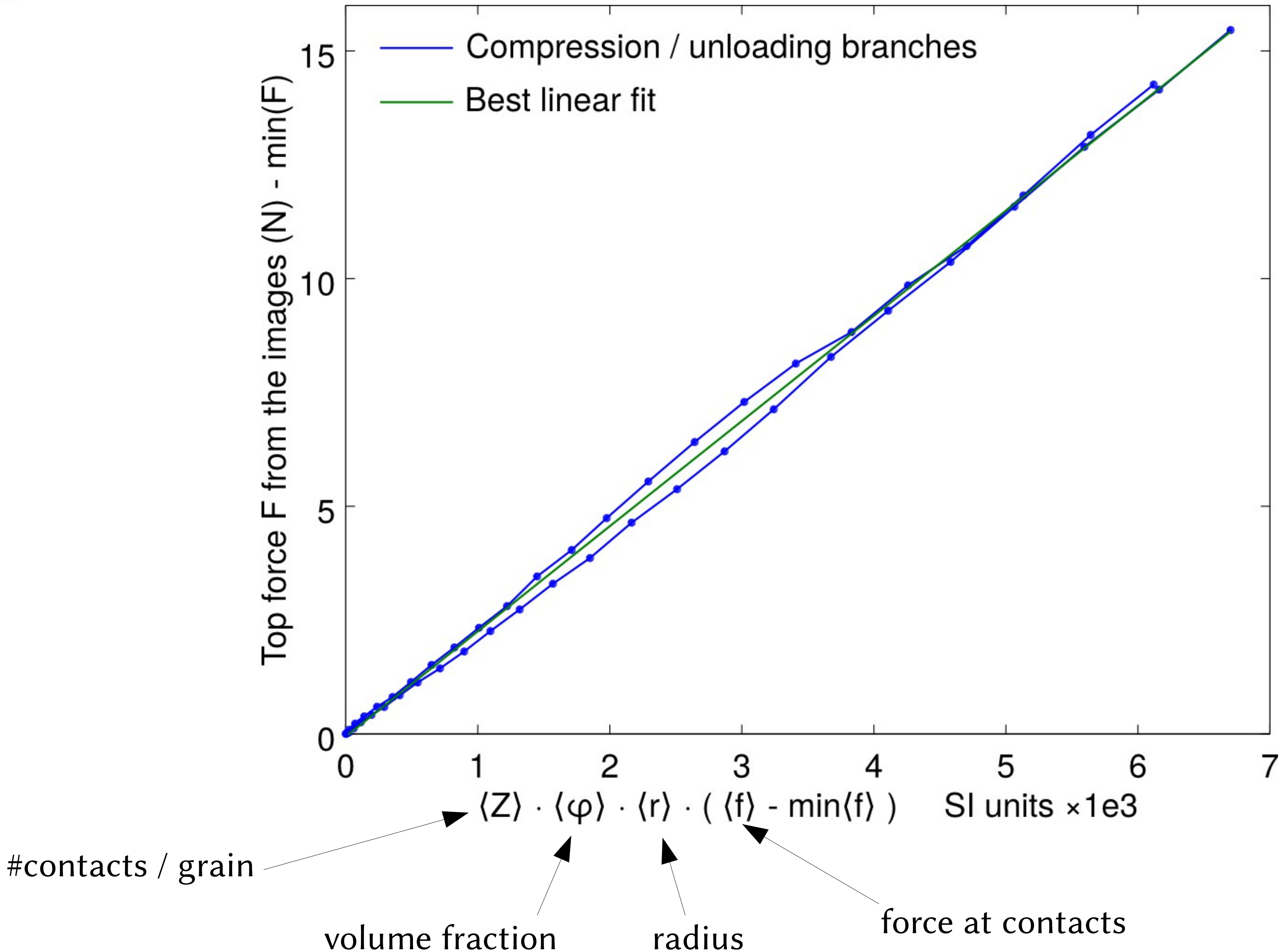
Weak contacts are missing and
below the experiment resolution !

The top plate does not touch the grains
= ignored scans

Non-Hertzian Packing response



A scaling holds, which is ...



… A MEAN STRESS TENSOR

A mean ⟨Z⟩ in the relation ⟹ some kind of isotropy between contacts

⟹ Use the isotropic pressure $p = \frac{1}{3}\mathrm{tr}S$ with S the stress tensor.

Without hydrostatic gradient (density match), the force on the top plate $F = p \cdot L \cdot W$

For a given volume element V_e : $S = \frac{1}{V_e} \sum_{c \in V_e} \mathbf{b} \otimes \mathbf{f_c}$

With **b** linking the grain centers and $\mathbf{f_c}$ the force vector at contact c.

Sphere approximation: the trace is simply the dot product: $\mathrm{tr}\,\mathbf{b} \otimes \mathbf{f_c} = \mathbf{b} \cdot \mathbf{f_c}$
and also ⟨b⟩=2⟨r⟩, with r=distance from center to contact for spheres.

The number of terms in the sum depends on the density of contacts ν. Incompressible grains ⟹ avg. volume ⟨V⟩ is constant. With φ the grain volume fraction: ν = ½ Z φ / ⟨V⟩

Replacing these terms by their averages over all contacts ⟹ $F \propto \langle Z \rangle \, \langle \varphi \rangle \, \langle r \rangle \, \langle f \rangle$

Note: subtracting min F and min ⟨f⟩ on both sides
in noisy experimental data for consistency with f=0 ⟹ F=0

Demo + Questions